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SEISMIC MIGRATION BY THE WCDP METHOD

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ABSTRACT

We describe realization and practical implementation of the wave analog of common depth point (WCDP) method. This method is based on the exact solution of inverse acoustic scattering, in the Born approximation, for determination of velocity heterogeneities, for reconstruction of velocity heterogeneities, using multiple overlapping data. The efficiency of the proposed algorithm is tested on synthetic and real data.

INTRODUCTION

The goal of this paper is to show possibility of the theory of inverse problems in construction of migration procedures for visualization of internal geometrical structures of the Earth. At present time the common depth point method and different modifications of the wave migration method [1] are the basic ones for multichannel seismic data processing. The WCDP method [2-3] is based on exact mathematical solution of the inverse scattering problem of acoustic waves, considered in the Born approximation, by multichannel overlapping data. It allows to take into account all wave exceptions of reflection and diffraction of seismic waves to increase, in resulting stack, the signal/noise ratio. We are focused, basically, on practical realization of the algorithm and its testing on synthetic and real data. We consider here 2D time variant of the WCDP method.

STATEMENT OF THE PROBLEM

Consider the case when acoustic waves originate from the pointwise impulsive source acting in the point $r_0=(x_0,0)$ and at the time $t=0$. We assume that the signal propagation $u(r,r_0,t)$, $r=(x,z)$, is governed in unbounded domain $(r,t)\in R^2\times R$ by the 2D wave equation [1]:

$$\Delta u - \frac{1}{c_0^2}(1+a(r))u_{tt} = \delta(r-r_0,t), \tag{1}$$

satisfied to the Cauchy data:

$$u\,|_{t<0} \equiv 0, \tag{2}$$

where c_0 is a background velocity. The response generated by this source is observed on the surface $z=0$:

$$u\,|_{z=0} \equiv u_0(x,x_0,t), (x,x_0,t) \in R^2 \times R_+, \tag{3}$$

where x is the position of a receiver on the surface $z=0$. The objective is to obtain information about variation of the propagation speed $a(r)$ from observations of the wave field $u_0(x,x_0,t)$.

SUMMATION FORMULA

The result of the solution of the inverse problem (1)-(3) in linear approximation is the following focusing operator [2-4]:

$$\beta(r) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\kappa \int_{-\infty}^{\infty} d\kappa_0 e^{i(\kappa+\kappa_0)x} \times \phi_c(z,\kappa,\kappa_0,\omega)\,\hat{u}_0(\kappa,\kappa_0,\omega). \tag{4}$$

The operator (4) allows us to calculate the visualization function $\beta(r)$, which is a local average of the required function $a(r)$ over domain with size of order of a sounding signal wavelength. In formula (4) the function $u_0(\kappa,\kappa_0,\omega)$ is the spectrum of recorded field,

$$\begin{aligned}\phi_c(z,\kappa,\kappa_0,\omega) &= \theta\left(k^2-\kappa^2\right)\theta\left(k^2-\kappa_0^2\right)|k|^{-1} \\ &\cdot\sqrt{k^2+\kappa\kappa_0+\sqrt{k^2-\kappa^2}+\sqrt{k^2-\kappa_0^2}} \\ &\cdot\exp\left\{-iz\left(\sqrt{k^2-\kappa^2}+\sqrt{k^2-\kappa_0^2}\right)\right\}\end{aligned} \tag{5}$$

is the kernel of the focusing operator, and $k=\omega c^{-1}$. Here $\theta(\cdot)$ is the usual Heaviside function, and c is *a priori* stacking velocity. The exponential factor in formula (5) provides a phase shift corresponding to the wave propagation time and similar to the phase shift in source-receiver coordinates in migration proposed in [5-6], but at same time their multipliers are the result of exact solution of the problem. The focusing operator (4)-(5) is the basis of the WCDP method.

The simple model of a seismic survey is an ensemble of data acquired using a zero-offset recording scheme, in which the source and receiver are located at the same point on the Earth´s surface. Unfortunately, in real seismic experiment it is not possible to use this simple shooting geometry because of persistent high amplitude reverberations associated with typical explosion, airgun, or vibrator sources used in seismic exploration. In order to realize the WCDP method in stacking formula (4)-(5), let us turn to common midpoint offset coordinates:

$$m = \frac{1}{2}(x + x_0), l = x - x_0. \tag{6}$$

Denote by μ and ν the frequency variables corresponding to *m* and *l*, correspondingly. Using the property of invariance of the wave phase we have $\kappa x+\kappa_0 x_0=\mu m+\nu l$. Thus we obtain from relations (6):

$$\kappa = \frac{\mu}{2} + \nu, \kappa_0 = \frac{\mu}{2} - \nu. \tag{7}$$

Let $U_0(m,l,t)$ be wave field in (m,l)-coordinates and $\hat{U}_0(\mu,\nu,\omega)$ be its spectrum. Using obvious equality $u_0(x,x_0,t)=U_0(m,l,t)$ and formulae (6) gives us:

$$\begin{aligned} u(x, x_0, t) &= U_0(\frac{x + x_0}{2}, x - x_0, t), \\ U_0(m, l, t) &= u_0(m + \frac{l}{2}, m - \frac{l}{2}, t). \end{aligned} \tag{8}$$

It is easy to establish the following connections between spectrums and reflected field:

$$\begin{aligned} \hat{u}_0(\kappa, \kappa_0, \omega) &= \hat{U}_0(\kappa + \kappa_0, \frac{1}{2}(\kappa - \kappa_0), \omega), \\ \hat{U}_0(\mu, \nu, \omega) &= \hat{u}_0(\frac{\mu}{2} + \nu, \frac{\mu}{2} - \nu, \omega). \end{aligned} \tag{9}$$

Let us introduce new variable:

$$q = c\left(\sqrt{k^2 - \kappa^2} + \sqrt{k^2 - \kappa_0^2}\right) \tag{10}$$

Using formulae (9) it is possible to obtain the final variant of the summation formulae (4)-(5):

$$\begin{aligned} \beta_c(m,t) = &\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} dq e^{iqt} \int_{-\infty}^{\infty} d\mu\, e^{i\mu m} \\ &\times \int_{-\infty}^{\infty} d\nu\, \phi_c(t,\mu,\nu,q)\hat{U}_0(\mu,\nu,q), \end{aligned} \tag{11}$$

with the kernel defined by formula:

$$\hat{\phi}_c(t, \mu, \nu, q) = \hat{\phi}_c(ct, \kappa, \kappa_0, \omega), \tag{12}$$

where (κ,κ_0) are connected with (μ,ν) by formula (7) and $\omega=\omega(q)$ is the solution of the equation (10). For numerical construction of the WCDP profile by formulae (4)-(5) or (11)-(12), the corresponding integrals are changed by integral sums with finite summation limits both with respect to time variable and with respect to spatial coordinates, too. There are two important advantages of formulae (11)-(12). First, the summations with respect to time frequency *q* and spatial frequency μ have the form of Fourier sums and by this reason the FFT-algorithm can be effective. The second reason is connected with choice of aperture of summation. Consider the general seismic plane giving on Fig. 1.

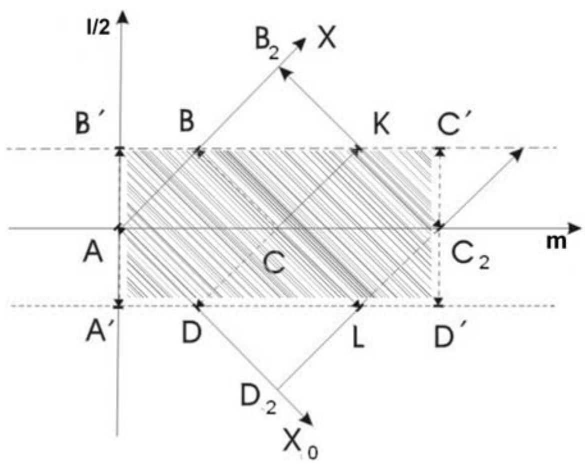


Figure1- Aperture summation WCDP in (x, x_0) and (m, l) -coordinates.

Each seismic trace of 2D profile is characterized by the coordinates (x,x_0) or (m,l), and can be represented as a point on the plane. Moreover, all the traces are located within band $/l/ \le l_{max}$, where l_{max} is the largest distance between source and receiver. The square *ABCD* on Fig. 1 corresponds to the aperture in (x,x_0)-coordinates, its diagonal *AC* forms summation base. Choice of the summation base is determined by depth of disposition and inclination of reconstructed boundaries. Its size is bounded by volume of RAM of computer too, that is connected with possibility of effective reconstruction of spatial spectrum of wave field with help of FFT algorithm. Increasing of summation base in (x,x_0)-coordinates leads to appearance of the domains (the triangles BB_2K and DLD_2) in aperture having no real seismic traces. For using of FFT algorithm we need to fill in by zero traces. It increases the volume of calculations but does not increase derivable profile comprehension. In "CMP-offset"-coordinates the aperture of summation is a *A'B'C'D* rectangle, in which the summation base is parallel to *m*-axes. The rectangle *A'B'C'D'* corresponds to the same base of summation, see Fig. 1. Such rectangle has only real recorded seismic trace or traces that can be reconstructed from them by reciprocity theorem.

SYNTHETIC DATA EXAMPLE

The model of a salt-dome structure shown on Fig. 2 is typical for many coastal sea-bed areas of the Atlantic ocean. Synthetic data for such a model were calculated for 521 explosion points by the solution to the acoustic wave equation with variable velocity and medium density with help of finite-difference approach [7]. The distance between explosion points and receivers was equal to 30m, the number of receivers per shot was equal to 96. The results of data processing are shown on Fig. 3 for the summation velocity equal to 1500m/s and 1600m/s. All the reflecting surfaces including inclined and subsalt reflectors are well reconstructed. It is necessary to mark dislocation of a subsalt reflector with respect to its real position is connected with a strong lateral velocity variation in the upper layers that does not taken into account in time migration by WCDP method. In addition, a strong multiple reflection is observed near to this boundary indicating to the necessity of preliminary data processing for its removal.

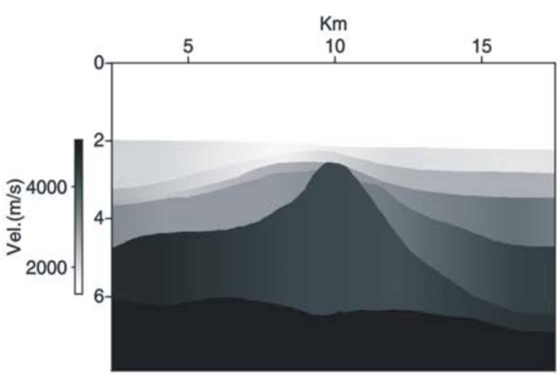
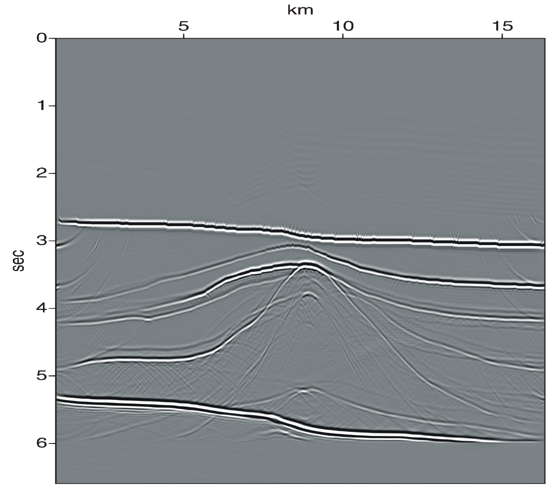
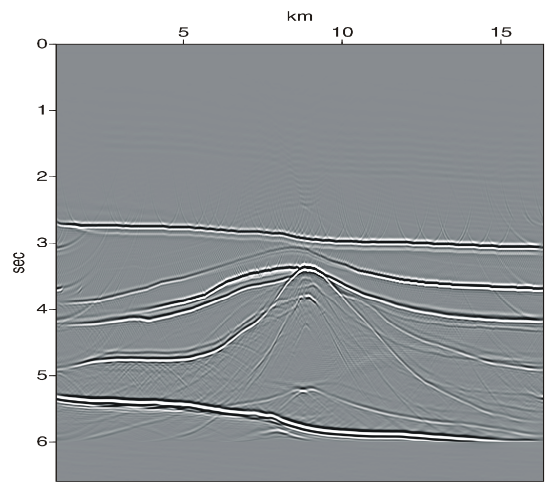


Figure 2 – A model of salt-diaper structure.



WCDP result, V_{input} = 1500 m/s.



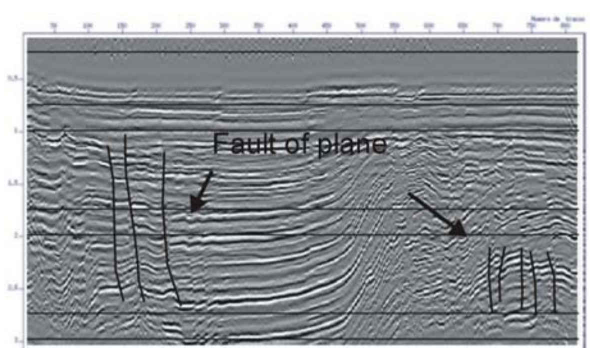
WCDP result, V_{input} = 1600 m/s.

Figure 3 – Note that all reflectors are good rebuild, though the last one move way their real positions due to strong lateral velocity variations.

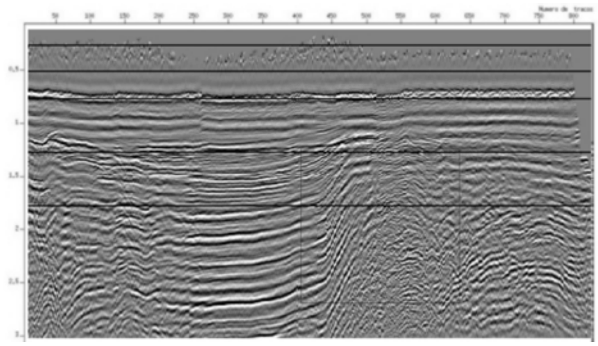
REAL DATA EXAMPLES

The WCDP method was applied for two different seismic maritime surveys and comparative analysis with NMO-CDP-stack was done in [3-4; 8]. The first example treats 2D line of Baikal Lake, Russia. The second one represents a 2D line extracted from 3D maritime survey denominated here as line A.

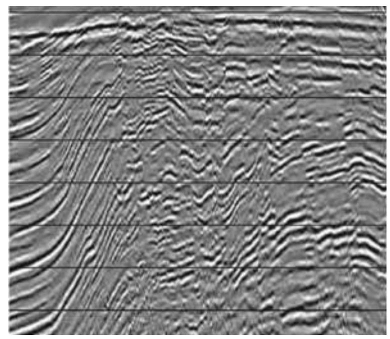
Fig. 4 shows the results and expanded view of the Baikal line reconstruction by WCDP and NMO-CDP-stack methods. Comparison of the two sections shows that WCDP method positions right the summation of both diffracted waves and reflected waves. Therefore, WCDP section is able to image small details of structures. NMO-CDP-stack section exhibits fault zone image distortions. In contrast, the WCDP image displays the structure clearly and offers superior imaging of the fault plane. It is possible by WCDP method to decrease the uncertainty and obtain new information about geological structures of offshore sediments, exhibit the nature of anomaly of wave field. According to details of Fig. 4, by NMO-CDP-stack method the diffractions of seismic waves were scattered giving a section with noise and low contents of information. Fig. 5 represents two seismic sections and their windows of migration by Kirchhoff and WCDP methods of offshore line A. Comparison of these results shows that the WCDP section gives us a better signal/noise ratio, continuity and position of reflection, definition of fault plane; and its possibility to improve details of some events in comparison with another section, kind of normal-slip fault, the sigmoid, the salt base and reflection under the salt. Although the Kirchhoff section has shown this reflection, the diffractions of seismic wave were scattered, see domain about time 3.0s and trace 600 (above salt line), which means the section with low information level. In the WCDP section we observe that diffractions were well collapsed, because its integral formulation, by Born approximation include the diffracted term not present on the Kirchhoff procedure based on theory of rays, but yet significant to solve great lateral variations of disturbances of velocities. Another aggravation is the necessity of the input of velocity field. This stage is dispensable in WCDP processing, as this part the uniform field velocity, which does not require velocity analysis *a priori*, and encounters internally an excellent value for the focalization velocity of reflection from each point of observation in seismic plane. By analysis of Fig. 5 it is evident that the method is effective, able to image the faults of region, which are absent or badly focalized in the corresponded Kirchhoff profile. The reason of superior quality of the WCDP method is due to migrating the seismic wave to the local of origin of secondary sources. As a result diffracted and reflected waves will be focalized and positioned in the local of its generation. The method is able to image well as the elements of diffraction just as smooth boundaries of great extension.



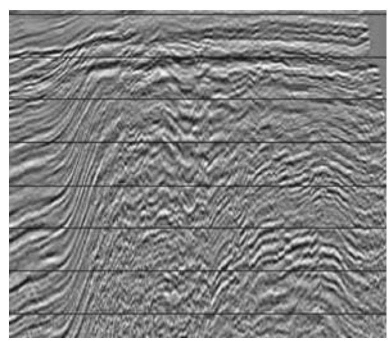
2D time WCDP section.



2D time NMO – CDP-stack.

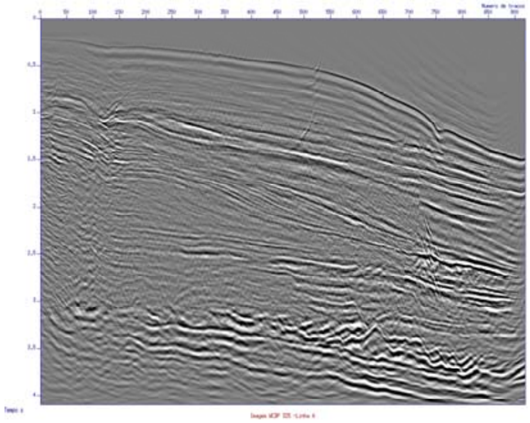


Expanded view of an area of fault zone in the WCDP section.

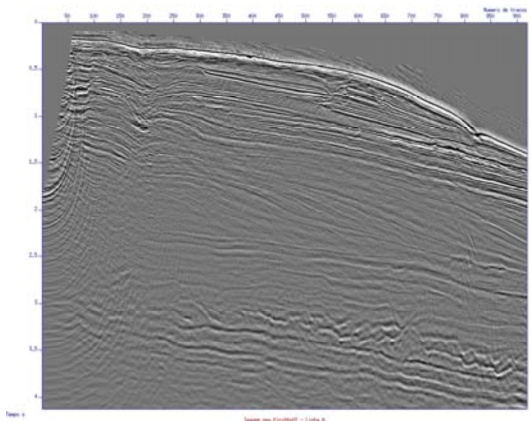


Expanded view of an area of fault zone in the NMO – CDP- stack.

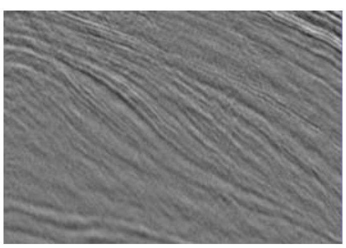
Figure 4 – A comparison of the two sections of Baikal Lake, Russia. NMO – CDP-stack exhibits distortions of fault zone image. In contrast, WCDP image displays the structure clearly and offers superior imaging of the fault plane as well.



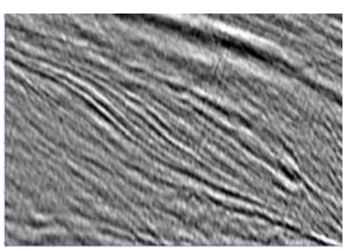
2D time WCDP section.



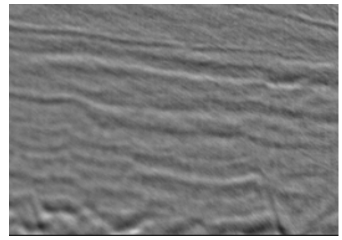
2D time Kirchhoff section.



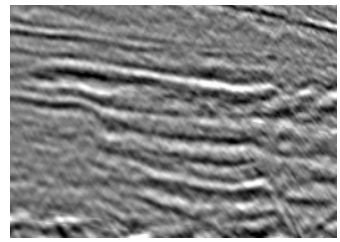
Expanded view of an area of sigmoid in Kirchhoff section.



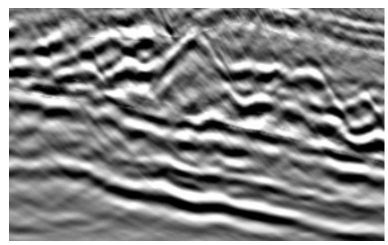
Expanded view of an area of sigmoid in WCDP section.



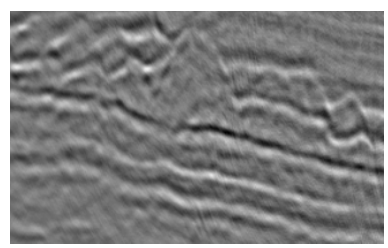
Expanded view of an area of fault zone in the Kirchhoff section.



Expanded view of an area of fault zone in the WCDP section.



Expanded view of an area of salt line and reflection under salt in the WCDP section.



Expanded view of an area of salt line and reflection under salt in the Kirchhoff section.

Figure 5 – A comparison of the two sections from a line A, Brazil. The Kirchhoff section exhibits distortions of fault zone image. In contrast, the WCDP section displays the structure clearly and offers superior imaging of the fault plane as well.

CONCLUSIONS

Investigation of exact solution of inverse acoustical problem in linear approximation with overlap data allows:

(1*) to take into account all wave singularities of seismic waves reflections and diffractions the most useful;*

(2) *to ensure useful signal accumulation*.

Practical realization of the WCDP method and its approbation on field data show high quality of reconstructed time profiles and good stability of the method for the choice of a priory velocity model.

Reservation of real amplitudes and undistorted wave packages on time profiles makes promising using of WCDP for investigation of dissipative properties of geological media.

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